

York University
EECS 2011E Fall 2015 – Problem Set 1
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This problem set will not be graded, but will help you understand asymptotic analysis of running times, and thus prepare for the midterm. You are free to work together on these if you prefer. Solutions will be posted Tuesday, Oct 20th. There are also problems at the end of Ch. 4 of the textbook if you would like additional practice.

1. Prove whether each of the following is true or false. x and y are real variables.

- 1) $\forall x \exists y x \cdot y = 5$
- 2) $\exists y \forall x x \cdot y = 5$
- 3) $\forall x \exists y x \cdot y = 0$
- 4) $\exists y \forall x x \cdot y = 0$
- 5) $\exists a \forall x \exists y [x = a \text{ or } x \cdot y = 5]$

2. Asymptotic Running Times

True or False? All logarithms are base 2. No justification is necessary.

- (a) $5n^2 \log n \in O(n^2)$
- (b) $4^{8n} \in O(8^{4n})$
- (c) $2^{10 \log n} + 100(\log n)^{11} \in O(n^{10})$
- (d) $2n^2 \log n + 3n^2 \in \Theta(n^3)$

3. Big-Oh Definition

Fill in the blanks:

$f(n) \in O(g(n))$ iff ____ $c > 0$, ____ $n_0 > 0$, such that ____ n ____ n_0 , $f(n)$ ____ $cg(n)$

4. Order the following functions by increasing asymptotic growth rate:

$$\begin{array}{ccc} 4n \log n + 2n & 2^{10} & 2^{\log n} \\ 3n + 100 \log n & 4n & 2^n \\ n^2 + 10n & n^3 & n \log n \end{array}$$

5. Prove that $n \log n - n$ is $\Omega(n)$.

6. Prove that if $d(n)$ is $O(f(n))$ and $e(n)$ is $O(g(n))$, then the product $d(n)e(n)$ is $O(f(n)g(n))$.

7. An evil king has n bottles of wine, and a spy has just poisoned one of them. Unfortunately, they don't know which one it is. The poison is very deadly; just one drop diluted even a billion to one will still kill. Even so, it takes a full month for the poison to take effect. Design a scheme for determining exactly which one of the wine bottles was poisoned in just one month's time while expending only $O(\log n)$ royal tasters. State your scheme briefly, in English.

8. Asymptotic Running Times

True or False? All logarithms are base 2. No justification is necessary.

- (a) $2^n \in \Omega(n^3)$
- (b) $3n^3 + 17n^2 \in O(n^3)$
- (c) $5n^2 \log n \in O(n^2)$
- (d) $2^{10 \log n} + 100(\log n)^{11} \in O(n^{10})$
- (e) $2n^2 \log n + 3n^2 \in \Theta(n^3)$

9. Show that n^2 is $\Omega(n \log n)$.